Modelling polaritons in a multimode cavity

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The Hamiltonian

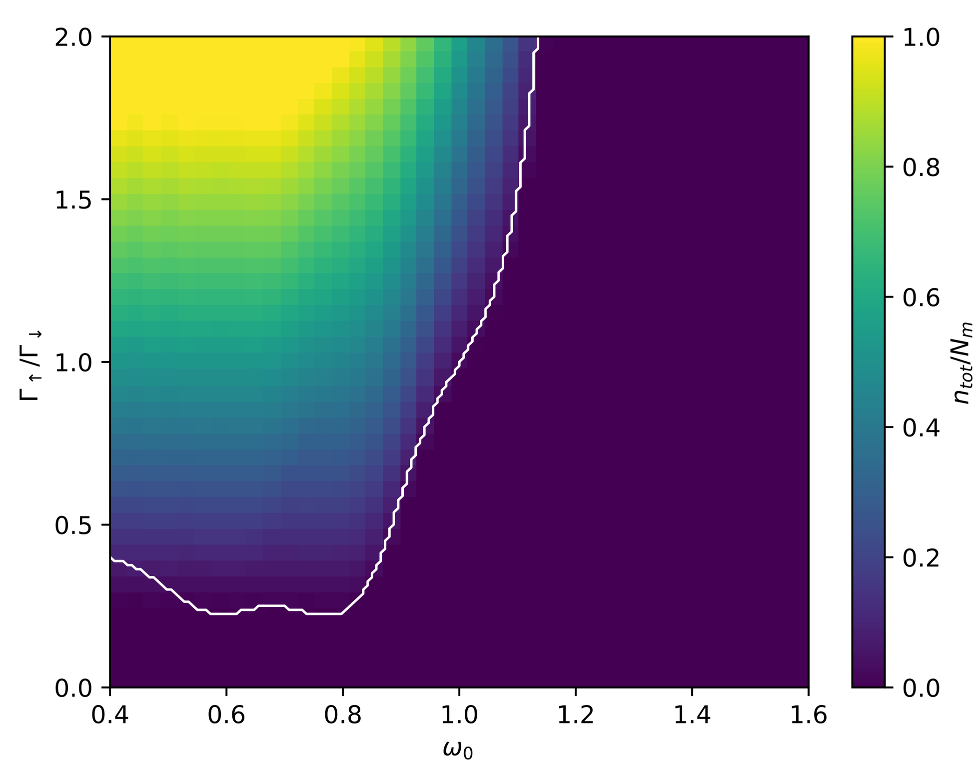
The Dicke-Holstein Hamiltonian with RWA and multiple photon modes:

$$H = \sum_{n}^{\mathcal{N}_m} \left[\frac{\varepsilon}{2}\sigma_n^z + \omega_v \left(b_n^\dagger b_n + \sqrt{S}\sigma_n^z (b_n^\dagger + b_n) \right) \right] + \sum_k^{\mathcal{N}_{ph}} \omega_k a_k^\dagger a_k + \sum_{k,n} g \left(\sigma_n^+ a_k e^{-ikr_n} + \sigma_n^- a_k^\dagger e^{ikr_n} \right)$$

Truncate phonon space to maximum $N-1$ phonons, resulting in the molecule being described by a $2N \times 2N$ Hermitian matrix. We use the *generalised Gell-Mann matrices* as a basis. The Hamiltonian then takes the form

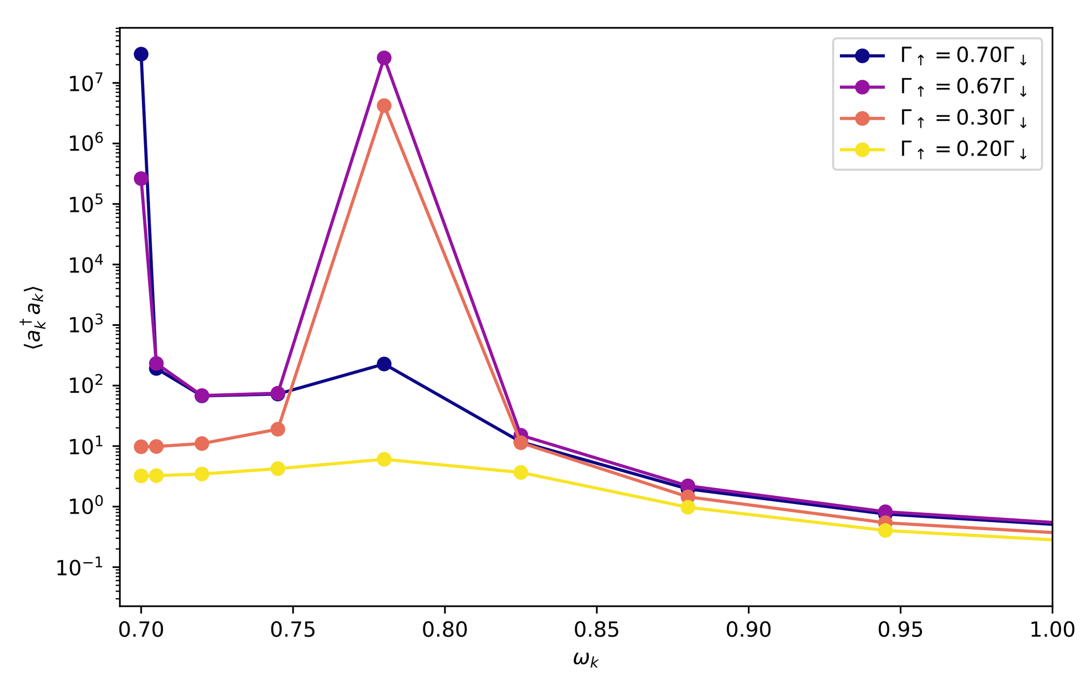
$$H = \sum_k \omega_k a_k^\dagger a_k + \sum_n \left[A_i + \sum_k \left(B_i a_k^\dagger e^{-ikr_n} + B_i^* a_k e^{ikr_n} \right) \right] \lambda_i^{(n)}$$

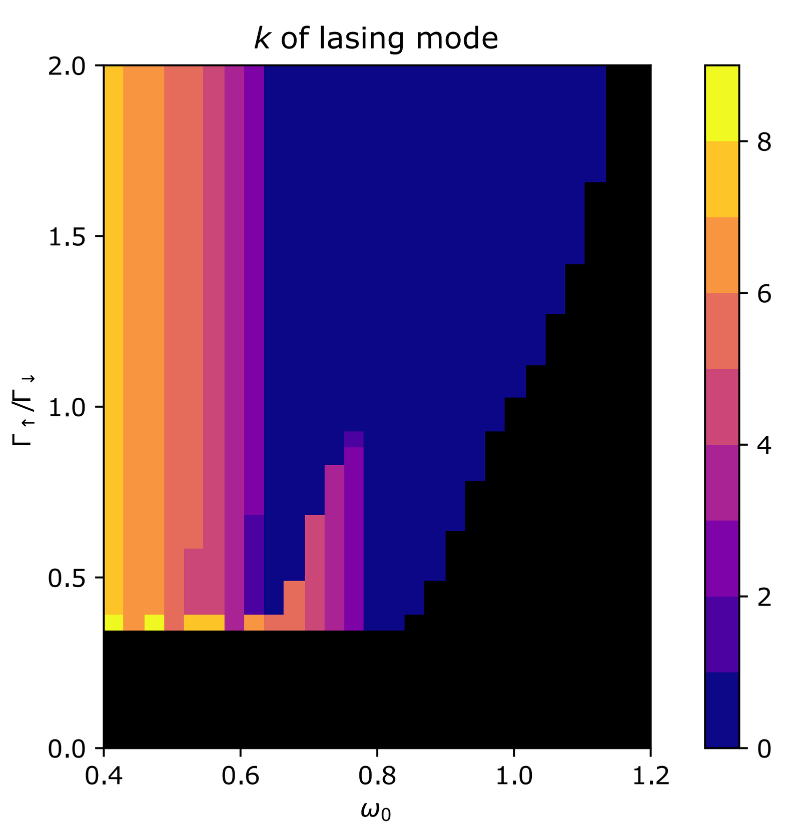
Steady state properties

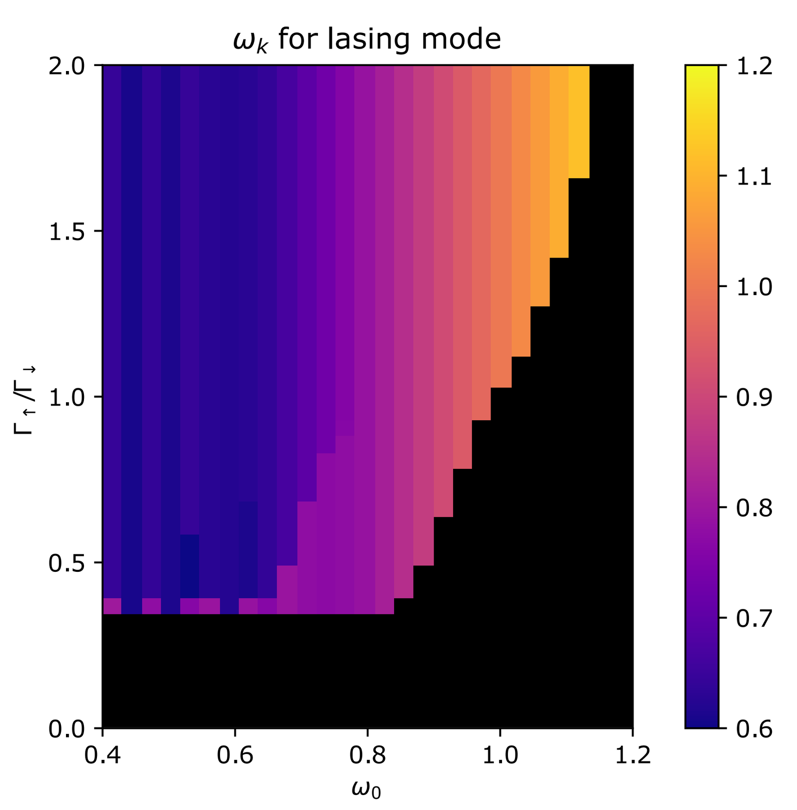


Parameters:

- $E_\rho = 10^5$
- $\Omega_R = 0.1$
- $S = 0.1$
- $\omega_v = 0.2$







Cumulants and symmetry

The cumulant average captures the deviation of the correlation of the total expression from the relevant linear combination of lower order terms. For example, second order cumulants are defined as:

$$\langle AB \rangle_c = \langle AB \rangle - \langle A \rangle \langle B \rangle.$$

Disregarding all cumulants → Mean field.

Why go beyond Mean field

- To capture thermalisation behaviours
- Needed to calculate fluoresence spectrum
- Mean field does not respect the symmetries of the Hamiltonian

We keep 2^{nd} order cumulants, but disregard all higher orders.

The system has $U(1)$ symmetry, so many averages vanish. We use the Lindblad master equation to write up a close set of equations of motion for

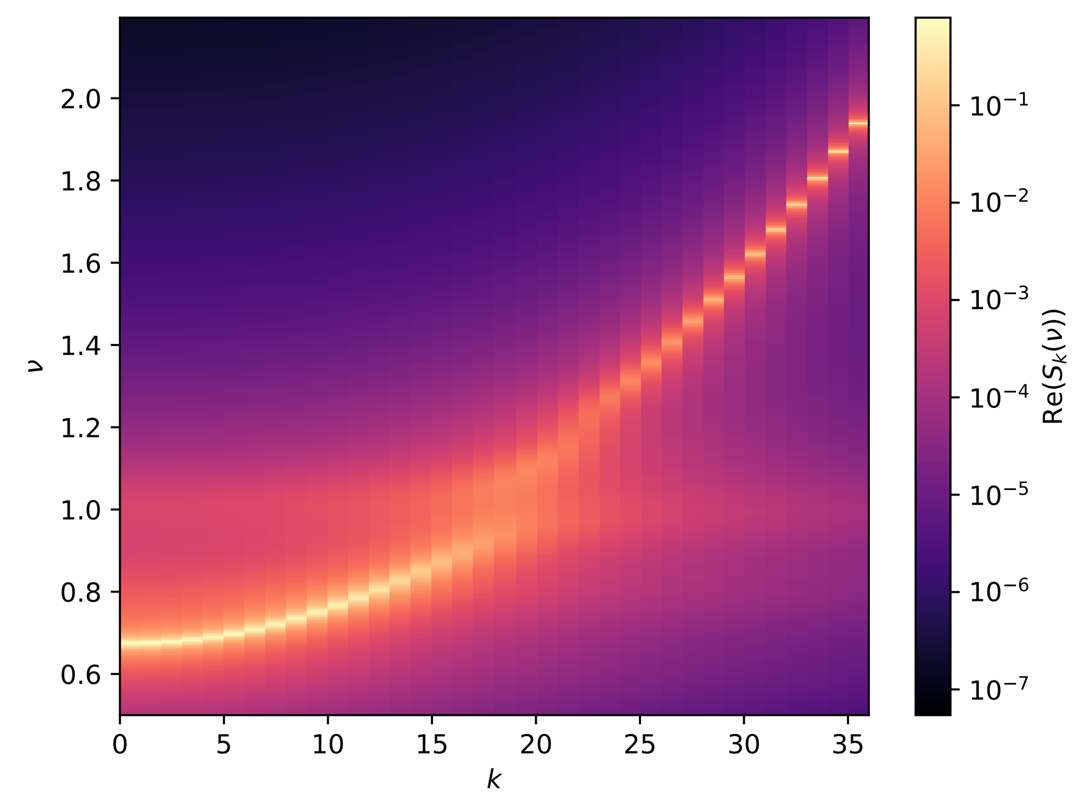
$$\langle \lambda_i^z \rangle, \langle a_k^\dagger a_k \rangle, \langle a_k \lambda_i^+ \rangle, \langle \lambda_i^+ \lambda_j^- \rangle$$

Fluorescence spectrum

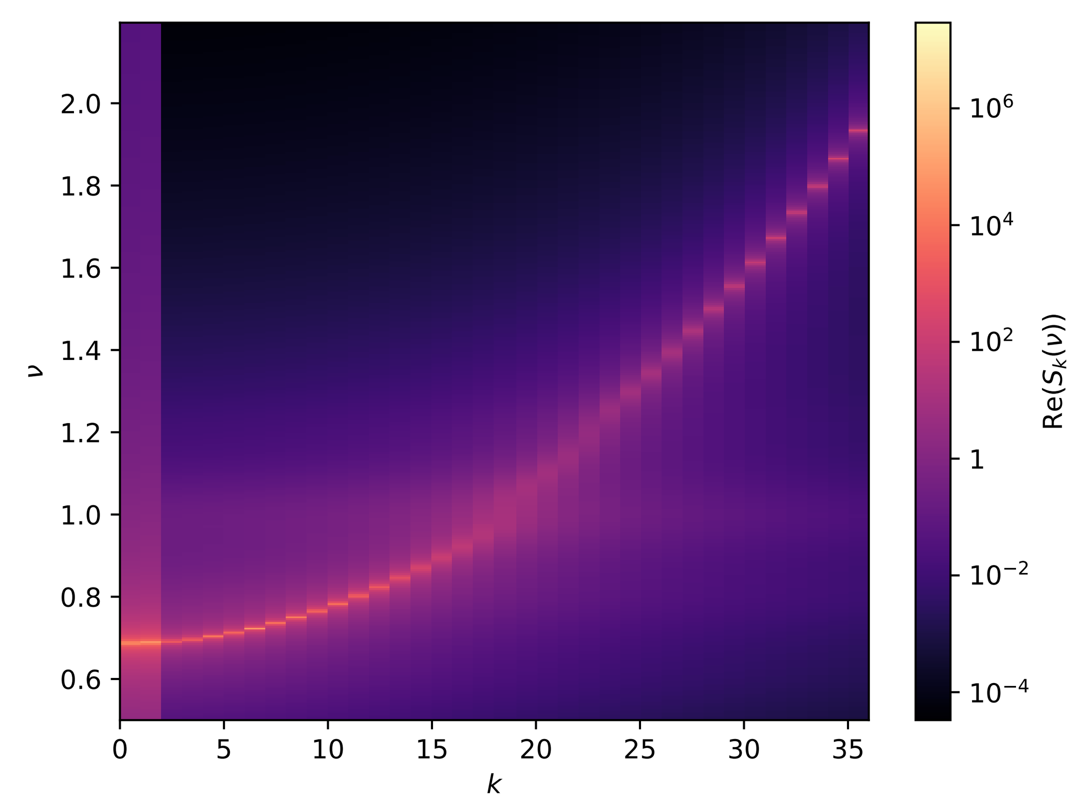
The fluorescence spectrum is the forward Fourier transform of the $g^{(1)}(t)$:

$$S_k(\nu) = \int_0^\infty dt \langle a_k^\dagger(t) a_k(0) \rangle e^{i\nu t}.$$

Assuming a steady state at $t = 0$ we can calculate the spectrum for our system. Example spectra:



- $\omega_0 = 0.7$
- $\Gamma_\uparrow = 0.001\Gamma_\downarrow$



- $\omega_v = 0.7$
- $\Gamma_\uparrow = \Gamma_\downarrow$