Emitter-centered EM modes as a minimal basis for multi-emitter QED in arbitrary dielectric environments

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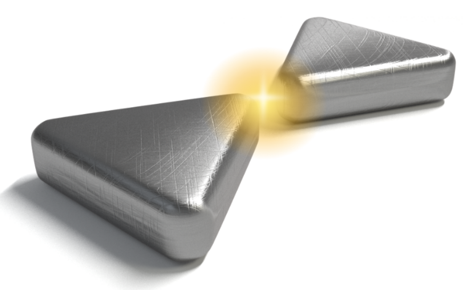
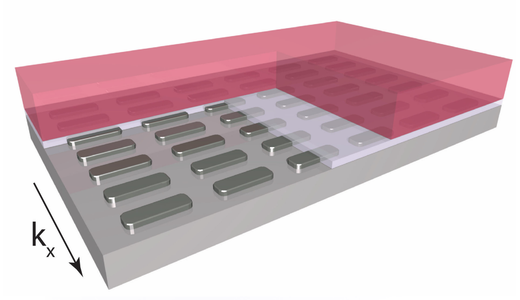
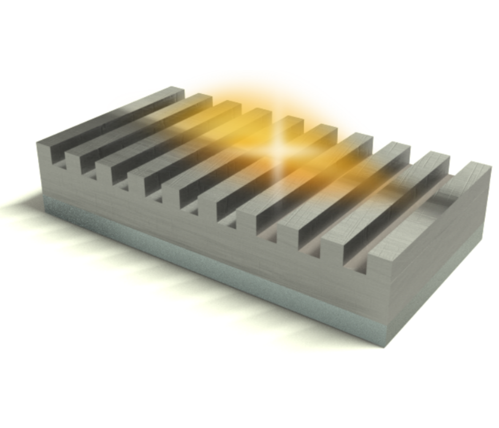
Introduction

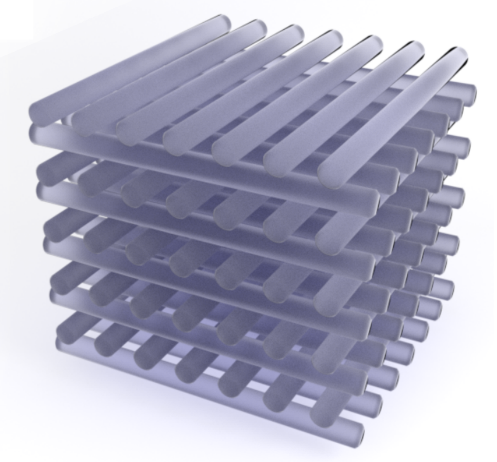
Emitter-cavity interactions

- Large variety of complex cavity designs available
- “Cavity”: any photonic structure that confines light
- Can enhance and control light-emitter interactions









- Common theoretical models (e.g., Jaynes-Cummings) treat a single (possibly lossy) quantized cavity mode
- Neglects the complex EM mode spectrum, and is only valid when the mode is spectrally separated from others and well-described by a Lorentzian spectral density (corresponding to exponential decay)
- Highly nontrivial to quantize EM modes in an arbitrary geometry with losses and material dispersion
- Formal solution: **Macroscopic QED** [1-4]

Macroscopic QED

- Separate system into **macroscopic** (EM properties given by dielectric function) and **microscopic** part (e.g., atom, molecule, or generally few-level quantum emitter)
- Formally represent material by an infinite set of local oscillators, $\hat{\mathbf{f}}(\mathbf{r},\omega)$, defined at each point in space, and for each frequency ($0<\omega<\infty$)
- Hamiltonian (no direct dipole-dipole interaction!):

$$H=\sum_i \hat{H}_i+\int_0^\infty \mathrm{d}\omega \int \mathrm{d}^3\mathbf{r}\,\hbar\omega\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)\hat{\mathbf{f}}(\mathbf{r},\omega) - \sum_i \hat{\mu}_i\mathbf{n}_i\cdot\left(\hat{\mathbf{E}}^{(+)}(\mathbf{r}_i)+\hat{\mathbf{E}}^{(-)}(\mathbf{r}_i)\right)$$

Here for simplicity: magnetic permeability μ = 1 [4]

- Electric field determined by **classical** EM Green’s function

$$\hat{\mathbf{E}}^{(+)}(\mathbf{r})=\int_0^\infty \mathrm{d}\omega\int \mathrm{d}^3\mathbf{r}'\,\mathbf{G}_e(\mathbf{r},\mathbf{r}',\omega)\cdot\hat{\mathbf{f}}(\mathbf{r}',\omega)$$

$$\mathbf{G}_e(\mathbf{r},\mathbf{r}',\omega)=i\frac{\omega^2}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0}\epsilon^I(\mathbf{r}',\omega)}\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)$$

- Dynamics of a single emitter at position $\mathbf{r}_0$ and direction $\mathbf{n}$ determined by **spectral density** (local density of states):

$$J(\omega)=\frac{\omega^2}{\pi\hbar\epsilon_0 c^2}\mathbf{n}\cdot\mathrm{Im}\,\mathbf{G}(\mathbf{r}_0,\mathbf{r}_0,\omega)\cdot\mathbf{n}$$

- Purcell factor: $F=J(\omega)/J_0(\omega)$

- **Challenge:** Formally, infinitely many modes everywhere in space and frequency. Direct use (e.g., by discretization) prohibitively expensive.
- **Possible solutions:**
  - **Integrate out EM part:**
    - Perturbation theory: Casimir-Polder, interatomic Coulomb decay, etc. [4,5]
    - Laplace transform / multiple scattering [6,7]
  - **Extract (few) quantized modes explicitly:**
    - Fit spectral density to sum of Lorentzians [7-13]
    - Explicit quantization of quasinormal modes [14]
- **In all cases: approximations necessary**

Emitter-centered modes

Single emitter

Question: What is the **minimal** set of quantized EM modes we need to represent the system **without approximations**?

Answer: Emitter-centered / collective / bright EM modes

- For each frequency ω, **one** superposition of the bare macroscopic QED modes $\hat{\mathbf{f}}(\mathbf{r},\omega)$ couples to the emitter.

$$\hat{B}(\omega)=\int \mathrm{d}^3\mathbf{r}\,\boldsymbol{\beta}(\mathbf{r},\omega)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega) \qquad \boldsymbol{\beta}(\mathbf{r},\omega)=\frac{\mathbf{n}\cdot\mathbf{G}_e(\mathbf{r}_0,\mathbf{r},\omega)}{\hbar\sqrt{J(\omega)}}$$

- **Quantized** / normalized correctly:

$$[\hat{B}(\omega),\hat{B}^\dagger(\omega')]=\delta(\omega-\omega')$$

- All other modes (orthogonal superpositions of the $\hat{\mathbf{f}}(\mathbf{r},\omega)$) uncoupled → drop from Hamiltonian:

$$H=\hat{H}_e+\int_0^\infty \mathrm{d}\omega\left[\hbar\omega\hat{B}^\dagger(\omega)\hat{B}(\omega)-\hat{\mu}_e\hbar\sqrt{J(\omega)}\left(\hat{B}(\omega)+\hat{B}^\dagger(\omega)\right)\right]$$

- **Electric field mode profile** available **(real!)**

$$\boldsymbol{\mathcal{E}}(\mathbf{r},\omega)=\frac{\omega^2}{\pi\epsilon_0 c^2}\frac{\mathrm{Im}\,\mathbf{G}(\mathbf{r},\mathbf{r}_0,\omega)\cdot\mathbf{n}}{\sqrt{J(\omega)}}$$

- **Minimal complete Hamiltonian** in an arbitrary dielectric environment.

Multiple emitters

- Modes associated to emitters *i* and *j* are **not orthogonal.**

$$[\hat{B}_i(\omega),\hat{B}_j^\dagger(\omega')]=S_{ij}(\omega)\delta(\omega-\omega')$$

$$S_{ij}=\int \mathrm{d}^3\mathbf{r}\,\boldsymbol{\beta}_i^*(\mathbf{r},\omega)\cdot\boldsymbol{\beta}_j(\mathbf{r},\omega)$$

- Overlap can be integrated analytically!

$$S_{ij}(\omega)=\frac{\omega^2}{\pi\hbar\epsilon_0 c^2}\frac{\mathbf{n}_i\cdot\mathrm{Im}\,\mathbf{G}(\mathbf{r}_i,\mathbf{r}_j,\omega)\cdot\mathbf{n}_j}{\sqrt{J_i(\omega)J_j(\omega)}}$$

- Orthogonalize: $\mathbf{V}(\omega)\mathbf{S}(\omega)\mathbf{V}^\dagger(\omega)=\mathbb{1}$

$$\hat{C}_i(\omega)=\sum_{j=1}^N V_{ij}(\omega)\hat{B}_j(\omega) \qquad [\hat{C}_i(\omega),\hat{C}_j^\dagger(\omega')]=\delta_{ij}\delta(\omega-\omega')$$

- Hamiltonian describes ***N* emitters** coupled to ***N* continua**

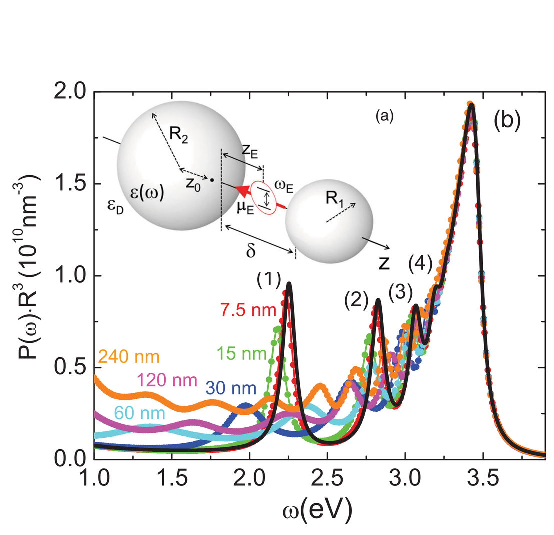
$$H=\sum_{i=1}^N\hat{H}_i+\int_0^\infty \mathrm{d}\omega\sum_{i=1}^N\hbar\omega\hat{C}_i^\dagger(\omega)\hat{C}_i(\omega) - \int_0^\infty \mathrm{d}\omega\sum_{i,j=1}^N\hat{\mu}_i\left(g_{ij}(\omega)\hat{C}_j(\omega)+g_{ij}^*(\omega)\hat{C}_j^\dagger(\omega)\right)$$

$$g_{ij}(\omega)=\hbar\sqrt{J_i(\omega)}W_{ij}(\omega)$$

- **Electric field operator:** $\hat{\mathbf{E}}^{(+)}(\mathbf{r})=\sum_{i=1}^N\int_0^\infty \mathrm{d}\omega\,\mathbf{E}_i(\mathbf{r},\omega)\hat{C}_i(\omega)$

$$\mathbf{E}_i(\mathbf{r},\omega)=\sum_{j=1}^N V_{ij}^*(\omega)\boldsymbol{\mathcal{E}}_j(\mathbf{r},\omega)$$

Example systems



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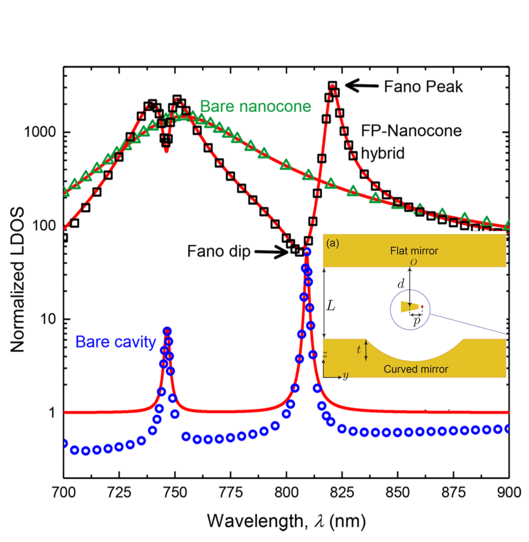

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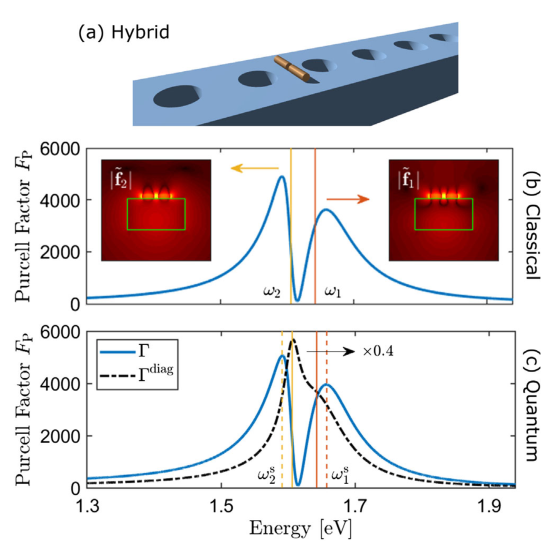

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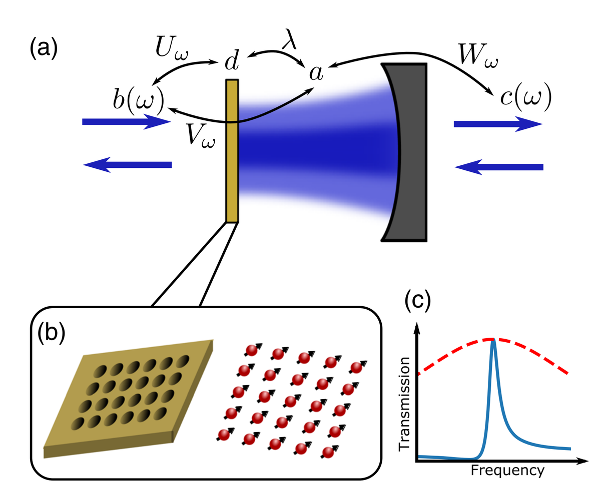

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Discussion

- Macroscopic QED can be used to construct an explicit set of **emitter-centered** EM modes that provide a minimal but complete basis for the quantum electrodynamics of a set of emitters in arbitrary dielectric environments.
- There are *N* photonic continua for *N* dipole emitters.
- The EM mode functions can be explicitly calculated and are real and divergenceless.
- System is **Hermitian** – no losses (price to pay: **continua**).
- While the formal construction requires the use of local oscillators at all points in space and frequencies, the final expressions **only** depend on the (imaginary part of) the **classical EM Green’s function** connecting the emitters between each other and/or measurement points.
- Approach can be used as starting point for either advanced numerical approaches, e.g., using tensor networks [15,16] or the cumulant expansion (poster M. Sánchez-Barquilla), or for more approximate treatments.
- **Outlook:**
  Beyond dipole approximation?
  Role of gauge (here: multipolar)?
  Formal approach to obtain simplified models?

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